

Operator terms

Zero-Order-Terms	
$\langle c \phi, \psi \rangle$	<code>Simple_ZOT</code> ($[c \in \mathbb{R}]$)
$\langle f(\vec{x}) \phi, \psi \rangle$	<code>CoordsAtQP_ZOT</code> ($f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v) \phi, \psi \rangle$	<code>VecAtQP_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $[f : \mathbb{R} \rightarrow \mathbb{R}]$)
$\langle f(v, \vec{x}) \phi, \psi \rangle$	<code>VecAndCoordsAtQP_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v) g(w) \phi, \psi \rangle$	<code>MultVecAtQP_ZOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$)
$\langle f(v, w) \phi, \psi \rangle$	<code>Vec2AtQP_ZOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $[f : (\mathbb{R})^2 \rightarrow \mathbb{R}]$)
$\langle f(v_1, v_2, v_3) \phi, \psi \rangle$	<code>Vec3AtQP_ZOT</code> ($v_1, v_2, v_3 \in \text{DOFVector}\langle \mathbb{R} \rangle$, $[f : (\mathbb{R})^3 \rightarrow \mathbb{R}]$)
$\langle f(\nabla v) \phi, \psi \rangle$	<code>FctGradient_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(\nabla v, \vec{x}) \phi, \psi \rangle$	<code>FctGradientCoords_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla v) \phi, \psi \rangle$	<code>VecAndGradAtQP_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla v, \vec{x}) \phi, \psi \rangle$	<code>VecGradCoordsAtQP_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla v, w) \phi, \psi \rangle$	<code>Vec2AndGradAtQP_ZOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$)
$\langle f(v, \nabla w) \phi, \psi \rangle$	<code>VecAndGradVecAtQP_ZOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v_1, v_2 \nabla v_3) \phi, \psi \rangle$	<code>Vec2AndGradVecAtQP_ZOT</code> ($v_1, v_2, v_3 \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla w_1, \nabla w_2) \phi, \psi \rangle$	<code>VecAndGradVec2AtQP_ZOT</code> ($v, w_1, w_2 \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, w, \nabla v, \nabla w) \phi, \psi \rangle$	<code>Vec2AndGrad2AtQP_ZOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(\{v_i\}_i) \phi, \psi \rangle$	<code>VecOfDOFVecsAtQP_ZOT</code> ($\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $f : \text{vec}\langle \mathbb{R} \rangle \rightarrow \mathbb{R}$)
$\langle f(\{\nabla v_i\}_i) \phi, \psi \rangle$	<code>VecOfGradientsAtQP_ZOT</code> ($\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $f : \text{vec}\langle \mathbb{R}^n \rangle \rightarrow \mathbb{R}$)
$\langle f(v, \{\nabla w_i\}_i) \phi, \psi \rangle$	<code>VecAndVecOfGradientsAtQP_ZOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $f : \mathbb{R} \times \text{vec}\langle \mathbb{R}^n \rangle \rightarrow \mathbb{R}$)
$\langle \partial_1 v_1 + \partial_2 v_2 + \partial_3 v_3 \rangle \phi, \psi \rangle$	<code>VecDivergence_ZOT</code> ($v_1, v_2, v_3 \in \text{DOFVector}\langle \mathbb{R} \rangle$)
$\langle f(\vec{x}, \{v_i\}, \{\nabla w_j\}) \phi, \psi \rangle$	<code>General_ZOT</code> ($\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $f : \mathbb{R}^n \times \text{vec}\langle \mathbb{R} \rangle \times \text{vec}\langle \mathbb{R}^n \rangle \rightarrow \mathbb{R}$)
$\langle f(\vec{x}, \vec{n}, \{v_i\}, \{\nabla w_j\}) \phi, \psi \rangle$	<code>GeneralParametric_ZOT</code> ($\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $f : \mathbb{R}^n \times \mathbb{R}^n \times \text{vec}\langle \mathbb{R} \rangle \times \text{vec}\langle \mathbb{R}^n \rangle \rightarrow \mathbb{R}$)

First-Order-Terms		
GRD_PHI	GRD_PSI	
$\langle c \vec{1} \cdot \nabla \phi, \psi \rangle$	$\langle c \vec{1} \phi, \nabla \psi \rangle$	<code>Simple_FOT</code> ($[c \in \mathbb{R}]$)
$\langle c \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle c \vec{b} \phi, \nabla \psi \rangle$	<code>Vector_FOT</code> ($\{b \in \mathbb{R}^n \mid i \in \mathbb{N}\}$, $[c \in \mathbb{R}]$)
$\langle f(v) \cdot \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle f(v) \cdot \vec{b} \phi, \nabla \psi \rangle$	<code>VecAtQP_FOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R} \rightarrow \mathbb{R}$, $\{b \in \mathbb{R}^n \mid i \in \mathbb{N}\}$)
$\langle f(v, w) \cdot \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle f(v, w) \cdot \vec{b} \phi, \nabla \psi \rangle$	<code>Vec2AtQP_FOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : (\mathbb{R})^2 \rightarrow \mathbb{R}$, $\{b \in \mathbb{R}^n \mid i \in \mathbb{N}\}$)
$\langle f(v_1, v_2, v_3) \cdot \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle f(v_1, v_2, v_3) \cdot \vec{b} \phi, \nabla \psi \rangle$	<code>Vec3AtQP_FOT</code> ($v_1, v_2, v_3 \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : (\mathbb{R})^3 \rightarrow \mathbb{R}$, $\{b^* \in \mathbb{R}^n \mid i \in \mathbb{N}\}$)
$\langle f(\vec{x}) \cdot \vec{1} \cdot \nabla \phi, \psi \rangle$	$\langle f(\vec{x}) \cdot \vec{1} \phi, \nabla \psi \rangle$	<code>CoordsAtQP_FOT</code> ($f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(\vec{x}) \cdot \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle f(\vec{x}) \cdot \vec{b} \phi, \nabla \psi \rangle$	<code>VecCoordsAtQP_FOT</code> ($f : \mathbb{R}^n \rightarrow \mathbb{R}$, $b^* \in \mathbb{R}^n$)
$\langle f(\vec{x}, v) \cdot \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle f(\vec{x}, v) \cdot \vec{b} \phi, \nabla \psi \rangle$	<code>FctVecAtQP_FOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$, $b^* \in \mathbb{R}^n$)
$\langle f(v, w, \nabla v) \cdot \vec{b} \cdot \nabla \phi, \psi \rangle$	$\langle f(v, w, \nabla v) \cdot \vec{b} \phi, \nabla \psi \rangle$	<code>Vec2AndGradAtQP_FOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $f : (\mathbb{R})^2 \times \mathbb{R}^n \rightarrow \mathbb{R}$, $b^* \in \mathbb{R}^n$)
$\langle F(\vec{x}) \cdot \nabla \phi, \psi \rangle$	$\langle F(\vec{x}) \phi, \nabla \psi \rangle$	<code>VecFctAtQP_FOT</code> ($F : \mathbb{R}^n \rightarrow \mathbb{R}^n$)
$\langle F(v) \cdot \nabla \phi, \psi \rangle$	$\langle F(v) \phi, \nabla \psi \rangle$	<code>VectorFct_FOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $F : \mathbb{R} \rightarrow \mathbb{R}^n$)
$\langle F(\nabla v) \cdot \nabla \phi, \psi \rangle$	$\langle F(\nabla v) \phi, \nabla \psi \rangle$	<code>VectorGradient_FOT</code> ($v \in \text{DOFVector}\langle \mathbb{R} \rangle$, $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$)
$\langle F(v, \nabla w) \cdot \nabla \phi, \psi \rangle$	$\langle F(v, \nabla w) \phi, \nabla \psi \rangle$	<code>VecGrad_FOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $F : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$)
$\langle F(\nabla v, \nabla w) \cdot \nabla \phi, \psi \rangle$	$\langle F(\nabla v, \nabla w) \phi, \nabla \psi \rangle$	<code>FctGrad2_FOT</code> ($v, w \in \text{DOFVector}\langle \mathbb{R} \rangle$, $F : (\mathbb{R}^n)^2 \rightarrow \mathbb{R}^n$)
$\langle F(\vec{x}, \{v_i\}, \{\nabla w_j\}) \cdot \nabla \phi, \psi \rangle$	$\langle F(\vec{x}, \{v_i\}, \{\nabla w_j\}) \phi, \nabla \psi \rangle$	<code>General_FOT</code> ($\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $F : \mathbb{R}^n \times \text{vec}\langle \mathbb{R} \rangle \times \text{vec}\langle \mathbb{R}^n \rangle \rightarrow \mathbb{R}^n$)
$\langle F(\vec{x}, \vec{n}, \{v_i\}, \{\nabla w_j\}) \cdot \nabla \phi, \psi \rangle$	$\langle F(\vec{x}, \vec{n}, \{v_i\}, \{\nabla w_j\}) \phi, \nabla \psi \rangle$	<code>GeneralParametric_FOT</code> ($\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $\text{vec}\langle \text{DOFVector}\langle \mathbb{R} \rangle \rangle$, $F : \mathbb{R}^n \times \mathbb{R}^n \times \text{vec}\langle \mathbb{R} \rangle \times \text{vec}\langle \mathbb{R}^n \rangle \rightarrow \mathbb{R}^n$)

Second-Order-Terms

$\langle c \cdot \nabla \phi, \nabla \psi \rangle$	<code>Simple_SOT</code> ($[c \in \mathbb{R}]$)
$\langle f(\vec{x}) \nabla \phi, \nabla \psi \rangle$	<code>CoordsAtQP_SOT</code> ($f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v) \nabla \phi, \nabla \psi \rangle$	<code>VecAtQP_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $[f : \mathbb{R} \rightarrow \mathbb{R}]$)
$\langle f(v, \vec{x}) \nabla \phi, \nabla \psi \rangle$	<code>VecAndCoordsAtQP_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, w) \nabla \phi, \nabla \psi \rangle$	<code>Vec2AtQP_SOT</code> ($v, w \in \text{DOFVector}(\mathbb{R})$, $[f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}]$)
$\langle f(\nabla v) \nabla \phi, \nabla \psi \rangle$	<code>FctGradient_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla v) \nabla \phi, \nabla \psi \rangle$	<code>VecAndGradAtQP_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla v, \vec{x}) \nabla \phi, \nabla \psi \rangle$	<code>VecGradCoordsAtQP_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle f(v, \nabla w) \nabla \phi, \nabla \psi \rangle$	<code>VecGrad_SOT</code> ($v, w \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$)
$\langle c \partial_j(\phi), \partial_i(\psi) \rangle$	<code>FactorIJ_SOT</code> ($i, j \in \mathbb{N}$, $c \in \mathbb{R}$)
$\langle f(\vec{x}) \partial_j(\phi), \partial_i(\psi) \rangle$	<code>CoordsAtQP_IJ_SOT</code> ($f : \mathbb{R}^n \rightarrow \mathbb{R}$, $i, j \in \mathbb{N}$)
$\langle f(v) \partial_j(\phi), \partial_i(\psi) \rangle$	<code>VecAtQP_IJ_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \rightarrow \mathbb{R}$, $i, j \in \mathbb{N}$)
$\langle f(v_1, v_2) \partial_j(\phi), \partial_i(\psi) \rangle$	<code>Vec2AtQP_IJ_SOT</code> ($v_1, v_2 \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, $i, j \in \mathbb{N}$)
$\langle B \nabla \phi, \nabla \psi \rangle$	<code>Matrix_SOT</code> ($B \in \mathbb{R}^{n \times n}$)
$\langle A(v) \nabla \phi, \nabla \psi \rangle$	<code>MatrixFct_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $A : \mathbb{R} \rightarrow \mathbb{R}^{n \times n}$, $[div^*]$)
$\langle B \cdot f(v, w) \nabla \phi, \nabla \psi \rangle$	<code>MatrixVec2_SOT</code> ($v, w \in \text{DOFVector}(\mathbb{R})$, $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, $B \in \mathbb{R}^{n \times n}$)
$\langle A(\nabla v) \nabla \phi, \nabla \psi \rangle$	<code>MatrixGradient_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $A : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, $[div^*]$)
$\langle A(v, \nabla v) \nabla \phi, \nabla \psi \rangle$	<code>VecMatrixGradientAtQP_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $A : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, $[div^*]$)
$\langle A(\nabla v, \vec{x}) \nabla \phi, \nabla \psi \rangle$	<code>MatrixGradientAndCoords_SOT</code> ($v \in \text{DOFVector}(\mathbb{R})$, $A : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, $[div^*]$)
$\langle A(\vec{x}, \{v_i\}, \{\nabla w_j\}) \nabla \phi, \nabla \psi \rangle$	<code>General_SOT</code> ($\text{vec}(\text{DOFVector}(\mathbb{R})), \text{vec}(\text{DOFVector}(\mathbb{R})), A : \mathbb{R}^n \times \text{vec}(\mathbb{R}) \times \text{vec}(\mathbb{R}^n) \rightarrow \mathbb{R}^{n \times n}$, $[div^*]$)
$\langle A(\vec{x}, \vec{n}, \{v_i\}, \{\nabla w_j\}) \nabla \phi, \nabla \psi \rangle$	<code>GeneralParametric_SOT</code> ($\text{vec}(\text{DOFVector}(\mathbb{R})), \text{vec}(\text{DOFVector}(\mathbb{R})), A : \mathbb{R}^n \times \mathbb{R}^n \times \text{vec}(\mathbb{R}) \times \text{vec}(\mathbb{R}^n) \rightarrow \mathbb{R}^{n \times n}$, $[div^*]$)

Comments

- All operators are listed in the files `ZeroOrderTerm.h`, `FirstOrderTerm.h` and `SecondOrderTerm.h`.
- The following definitions/shortcuts are used to reduce typing: L_2 -Scalar product: $\langle \cdot, \cdot \rangle$, trialfunction: ϕ , testfunction ψ , coefficients $c \in \mathbb{R}$, $\vec{1}, \vec{b}, \vec{x} \in \mathbb{R}^n$, with $(\vec{1})_i = 1$, $B \in \mathbb{R}^{n \times n}$, functors $f : (\dots) \rightarrow \mathbb{R}$, $F : (\dots) \rightarrow \mathbb{R}^n$ and $A : (\dots) \rightarrow \mathbb{R}^{n \times n}$.
- Some mathematical notations are used to describe data-structures: \mathbb{R} means `double`, \mathbb{R}^n means `WorldVector<double>` and $\mathbb{R}^{n \times n}$ means `WorldMatrix<double>`.
- f, F, A can be implemented as `(*)AbstractFunction(ReturnType, InputType1, InputType2, ...)`, where `(*)` $\in \{\emptyset, \text{Binary}, \text{Tertiary}, \text{Quart}\}$ depending on the number of input arguments.
- The data-structure `DOFVector<*>` is always a pointer to a `DOFVector`.
- Optional arguments are depicted in square brackets `[*]`, where constants $c = 1$ by default, functions are `NULL`-pointers by default and are treated as identity functors or simple multiplication functors.
- Alternative argument of the form $\{b \in \mathbb{R}^n \mid i \in \mathbb{N}\}$ mean in the second case: $\vec{b} := \vec{e}_i$.
- The expression $b^* \in \mathbb{R}^n$ means a pointer to a `WorldVector<double>`.
- The argument $div := \mathbb{R}^{n \times n} \rightarrow \mathbb{R}^n$ is only interesting for error estimators and optional. Is should implement the divergence of the matrix function in the operator.
- The argument `vec(*)` should be implemented as `std::vector(*)`.
- In the last Second-Order-Operator `GeneralParametric_SOT`, the second argument \vec{n} to the functor is the elementnormal, especially for surface meshes.